

Discovering Math: Concepts in Number Theory

Teacher's Guide

Grade Level: 10–12

Curriculum Focus: Mathematics

Lesson Duration: Three class periods

Program Description

Concepts in Number Theory – Explore planetary orbits, circadian rhythms, time dilation, intergalactic wormholes, and other phenomena that quantify time. Discover how the Maya relied on math. And examine the importance of rational numbers in physics, complex numbers in the real world, and logarithms and exponents in nature.

Onscreen Questions

- What does it mean to say something grows exponentially?
 - How has the development of complex numbers helped in the study of electricity?
 - How was the ancient Maya numbering system different from other systems?
 - Name the type of system the Maya developed.
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Lesson Plan

Student Objectives

- Understand Maya achievements in mathematics.
- Understand the Maya calendar.
- Learn how to convert Maya numbers to decimal numbers, and vice versa.
- Learn basic Maya arithmetic: addition, subtraction, multiplication, and division.

Materials

- *Discovering Math: Concepts in Number Theory* video
- Computer with Internet access
- Print resources about the history of Maya mathematics

Procedures

1. Have students research the Maya culture and create a time line of Maya civilization using print and Web resources. The following Web sites are a good starting point:

- Civilization.ca - Mystery of the Maya - civilization timeline
<http://www.civilization.ca/civil/maya/mmc09eng.html>
- Mayan World
<http://www.mayan-world.com/time.htm>
- The Maya Civilization, The Time-Line
http://www.mexconnect.com/mex_/travel/ldumois/maya/ldmayatimeline.html

Have the students create a bulletin board from their time lines.

2. Have students research the Maya calendar using print and Web resources. The following Web sites are a good starting point:

- Maya calendar
http://en.wikipedia.org/wiki/Maya_calendar
- The Classic Maya Calendar and Day Numbering System
<http://www.eecis.udel.edu/~mills/maya.html>
- Maya Calendar – Yucatan's Maya World Studies Center
<http://www.mayacalendar.com/>
- Maya Calendar
<http://www.michiellb.nl/maya/calendar.html>

3. Have students summarize their research in a one-page report.
4. Group students in pairs to share their reports and answer any questions. Then have each student present the main points of their partner's report to the class, including at least two interesting facts.
5. Review the decimal number system, place value, and expanded notation. Show students how to convert base 20 numbers with decimal digits to base 10 numbers. Have students convert several base 20 numbers to base 10 numbers on their own.
6. To familiarize the students with Maya numbers, have the students write the Maya numbers from 1 to 26. Show students examples of converting Maya numbers to decimal numbers and converting decimal numbers to Maya numbers.
Remind students of the three rules of addition shown in the video: a dot equals one, five dots equals one bar, and four bars equals one dot in the next place
Show students examples of adding Maya numbers and allow them time for practice.
7. Show students examples of subtracting Maya numbers and provide students time for practice.



8. Review the properties of multiplication as they apply to Maya numbers; that is, the identity property, the zero property, and the commutative, associative, and distributive properties. Show students examples of multiplying Maya numbers and allow them time for practice.
9. Show students examples of dividing Maya numbers and allow them time for practice.

Assessment

Use the following three-point rubric to evaluate students' work during this lesson.

- **3 points:** Students were highly engaged in class discussions; produced complete reports, including all of the requested information; clearly demonstrated the ability to convert between Maya and decimal numbers, and showed a complete understanding of Maya arithmetic.
- **2 points:** Students participated in class discussions; produced an adequate report, including most of the requested information; satisfactorily demonstrated the ability to convert between Maya and decimal numbers, and showed a satisfactory understanding of Maya arithmetic.
- **1 point:** Students participated minimally in class discussions; created an incomplete report with little or none of the requested information; were not able to convert between Maya and decimal numbers or adequately perform Maya arithmetic.

Vocabulary

decimal number system

Definition: A positional system of numeration that uses decimal digits and a base of 10

Context: The decimal number system is the most common numeral system used around the world.

expanded notation

Definition: A numeral expressed as a sum of the products of each digit and its place value

Context: Expanded notation is used when converting between the Maya number system and the decimal number system.


glyph

Definition: A symbolic figure or a character usually incised or carved in relief

Context: The Maya used glyphs to represent days and months in their calendar.

place value

Definition: The value of a digit as determined by its position in a number

Context: In the Maya number , the top dot is in the 20^2 (four hundreds) place, the shell is in the 20^1 (twenties) place, and the bottom dot is in the 20^0 (ones).

Maya number system

Definition: A positional system of numeration that uses a base of 20 and three symbols: zero (a shell-shaped glyph), one (a dot), and five (a bar)

Context: Advanced features of the Maya number system are the zero, represented by a shell, and the place value system.

vigesimal number system

Definition: Any positional system of numeration that uses a base of 20

Context: The Maya number system is a vigesimal number system.

Academic Standards

National Council of Teachers of Mathematics (NCTM)

The National Council of Teachers of Mathematics provides guidelines for teaching mathematics in grades K–12 to promote mathematical literacy. To view the standards, visit this Web site:

<http://standards.nctm.org/document/chapter3/index.htm>

This lesson plan addresses the following thematic standards:

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems

Mid-continent Research for Education and Learning (McREL)

McREL's Content Knowledge: A Compendium of Standards and Benchmarks for K-12 Education addresses 14 content areas. To view the standards and benchmarks, visit

<http://www.mcrel.org/compendium/browse.asp> .

This lesson plan addresses the following national standards:

- Mathematics: Understands and applies basic and advanced properties of the concepts of numbers; Understands and applies basic and advanced properties of functions and algebra; Understands the general nature and uses of mathematics
- Science: Physical Science: Understands the structure and properties of matter; Understands the sources and properties of energy
- World History: Understands Maya achievements in mathematics
- Historical Understanding: Understands the historical perspective

Support Materials

Develop custom worksheets, educational puzzles, online quizzes, and more with the free teaching tools offered on the Discoveryschool.com Web site. Create and print support materials, or save them to a Custom Classroom account for future use. To learn more, visit

- <http://school.discovery.com/teachingtools/teachingtools.html>



DVD Content

This program is available in an interactive DVD format. The following information and activities are specific to the DVD version.

How To Use the DVD

The DVD starting screen has the following options:

Play Video – This plays the video from start to finish. There are no programmed stops, except by using a remote control. With a computer, depending on the particular software player, a pause button is included with the other video controls.

Video Index – Here the video is divided into five parts (see below), indicated by video thumbnail icons. Watching all parts in sequence is similar to watching the video from start to finish. Brief descriptions and total running times are noted for each part. To play a particular segment, press Enter on the remote for TV playback; on a computer, click once to highlight a thumbnail and read the accompanying text description and click again to start the video.

Curriculum Units – These are specially edited video segments pulled from different sections of the video (see below). These nonlinear segments align with key ideas in the unit of instruction. They include onscreen pre- and post-viewing questions, reproduced below in this Teacher's Guide. Total running times for these segments are noted. To play a particular segment, press Enter on the TV remote or click once on the Curriculum Unit title on a computer.

Standards Link – Selecting this option displays a single screen that lists the national academic standards the video addresses.

Teacher Resources – This screen gives the technical support number and Web site address.

Video Index

I. Exploring Exponents (5 min.)

Bacteria grow exponentially. Learn how to calculate bacteria growth using exponential functions.

II. Rational Numbers (6 min.)

Learn how rational numbers can be used to convert back and forth between degrees Celsius and degrees Fahrenheit. Learn how the boiling point of water depends on the altitude.

III. Logarithmic Spirals (5 min.)

The pattern of a sunflower, a picture of a hurricane, galaxy spirals, waves, and certain architectural designs are examples of logarithmic spirals. Learn equations for logarithmic spirals and how to graph them.

IV. Complex Numbers (6min.)

Learn how complex numbers help to express electrical concepts.

V. Math and Culture: The Ancient Maya (26 min.)

The ancient Maya number system used a base 20 system and three symbols: the dot, the bar, and the shell. Learn how to write decimal numbers as Maya numbers and vice versa. Learn how the Maya calendar works and how to add, subtract, multiply, and divide Maya numbers.



Curriculum Units

1. Exponential Growth

Pre-viewing question

Q: How are exponents used in everyday life?

A: Answers will vary.

Post-viewing question

Q: Starting with one bacterium, how many will there be after three days if the bacteria divide in two every half hour?

A: In three days there are $3 \times 2 \times 24 = 144$ half hours.

$$y = 2^{144}$$

$$y = 2.2 \times 10^{43}$$

2. Exponential Functions

Pre-viewing question

Q: Besides bacteria, what else can grow exponentially?

A: Answers will vary.

Post-viewing question

Q: What is the general form for an exponential function?

A: $f(x) = a \cdot bx$

3. Rational Functions and Temperature Conversion

Pre-viewing question

Q: How does temperature affect your life?

A: Answers will vary.

Post-viewing question

Q: How do you convert degrees Celsius to degrees Fahrenheit and the reverse?

A: To convert degrees Celsius to degrees Fahrenheit, use the formula $^{\circ}F = \left(\frac{9}{5} \times T_C\right) + 32$, where T_C is degrees Celsius. To convert degrees Fahrenheit to degrees Celsius, use the formula $^{\circ}C = \frac{5}{9}(T_F - 32)$, where T_F is degrees Fahrenheit.

4. Rational Functions and Water's Boiling Point

Pre-viewing question

Q: How does altitude affect temperature?

A: Answers will vary.

Post-viewing question

Q: Mt. McKinley in Alaska has an altitude of 20,320 feet. How do you find the boiling point of water on that peak?



A: Method 1: Use the formula $B = 212 - \frac{37x}{20}$, where x is thousands of feet.

$$\frac{20,320}{1000} = 20.32$$

$$B = 212 - \frac{37x}{20}$$

$$B = 212 - \frac{37(20.32)}{20} = 212 - 37.6 = 174.4^\circ F$$

Method 2: The boiling point decreases at a rate of $-0.185^\circ F$ per 100 feet.

$$B = 212 - 0.185 \times \frac{20,320}{100} = 212 - 37.6 = 174.4^\circ F$$

5. Logarithmic Spirals

Q: Which patterns in nature do you think are beautiful?

A: Answers will vary.

Post-viewing question

Q: What is the difference between the graph of $r = e^{0.2\theta}$ and the graph of $r = e^{0.35\theta}$?

A: The graph of $r = e^{0.2\theta}$ is a tighter spiral than the graph of $r = e^{0.35\theta}$.

6. Complex Numbers

Pre-viewing question

Q: What is an example of an equation that has no real number solutions?

A: Answers will vary.

Post-viewing question

Q: What is a complex number?

A: A complex number is a number of the form $a + bi$, where a and b are real numbers, and $i = \sqrt{-1}$.

7. Complex Numbers and Electricity

Pre-viewing question

Q: How does electricity affect your life?

A: Answers will vary.

Post-viewing question

Q: How do you add and multiply complex numbers?

A: To add complex numbers, add the real parts and add the imaginary parts:

$$(a + bi) + (c + di) = (a + b) + (c + d)i$$

To multiply complex numbers, use FOIL:

$$\begin{aligned}(a + bi)(c + di) &= ac + adi + bci + bdi^2 \\ &= ac + adi + bci + bd(-1) \quad (i^2 = -1) \\ &= (ac - bd) + (ad + bc)i\end{aligned}$$



8. Maya Number System

Pre-viewing question



Q: Describe a number system different than the decimal number system.

A: Answers will vary.

Post-viewing question

Q: Compare and contrast the decimal number system and the Maya number system.

A: The decimal number system has the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 and has a place value of 10.

The Maya number system has the digits , •, and —, and has a place value of 20. The values of the Maya digits in the decimal system are as follows:  = 0, • = 1, and — = 5.

9. Maya Calendar

Pre-viewing question

Q: Why do we have a calendar system?

A: Answers will vary.

Post-viewing question

Q: Describe the Maya calendar.

A: Answers will vary.

10. Addition of Maya Numbers

Pre-viewing question

Q: How do you add two three-digit decimal numbers?

A: Answers will vary.

Post-viewing question

Q: Why is it easier to add Maya numbers than it is to add decimal numbers?

A: Answers will vary.

11. Subtraction of Maya Numbers

Pre-viewing question

Q: How do you subtract two numbers when you need to regroup?

A: Answers will vary.

Post-viewing question

Q: How do you subtract two Maya numbers?

A: Answers will vary.

12. Multiplication and Division of Maya Numbers

Pre-viewing question

Q: How do you multiply two 3-digit numbers?

A: Answers will vary.

Post-viewing question

Q: Why is it easier to multiply Maya numbers than it is to multiply decimal numbers?

A: Answers will vary.